Algebraic Content and Pedagogical Knowledge of Sixth Grade Mathematics Teachers: Through Document Analysis

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Abstract
Research indicates that mathematical content and pedagogical knowledge of teachers play a crucial role in teaching mathematical concepts in a way that helps students develop conceptual understanding of the subject matter. Conceptual understanding has been linked with the ability of the teacher to explain certain concepts. It is proven that teachers who have a deep understanding of the content and pedagogy foster better student performance as they will employ better instructional practices than teachers who lack an in-depth understanding of the content and pedagogy. Documents and written materials such as lesson plans and notes can provide rich information and evidence in qualitative studies. Research shows that pedagogical knowledge of teachers could be observed through teachers' lesson plans and lesson notes. This paper examined algebraic content and pedagogical of sixth grade mathematics teachers through analysis of lesson plans, lesson notes, textbooks, and reference materials they used. Content of all algebra lesson plans and notes (N=80) and prescribed textbooks (N=3) were analyzed. Results showed that some of the algebraic concepts explained in the textbooks were incorrect. As teachers failed to detect these incorrect concepts it raised serious questions with regard to algebraic content and pedagogical knowledge of sixth grade mathematics teachers. Moreover, analysis of the textbooks showed that the focus of the textbooks was on procedural fluency rather than conceptual understanding. Last, but not least, teachers followed these textbooks word for word blindly indicating that they not only lacked content knowledge but also pedagogical knowledge to teach algebra.

Keywords: content knowledge, pedagogical knowledge, algebra, document analysis

Introduction and Background

A baseline survey conducted by UNICEF and NIE to assess student performance during 2012 and 2013 in the Maldives showed that algebra is the lowest performing area of mathematics, with only 27.1% students obtaining the correct answers to algebra questions (UNICEF & NIE, 2014). Algebra is a fundamental topic in mathematics that lays foundation for further studies in mathematics and quantitative sciences. Interviews with the local project manager of the baseline study, mathematics teacher educators, heads of fourteen 1-10 schools, and heads of teacher training institutions gave rise to serious questions regarding the algebraic content and pedagogical knowledge of the teachers, and, therefore, the teachers' ability to teach the subject. In the Maldives, algebra is first introduced in sixth grade, and teachers who introduce algebra are responsible for laying a solid foundation on which students can later construct their algebraic understanding. It is proven that teachers who have a deep understanding of the content and pedagogy foster better student performance as they will employ better instructional practices than teachers who lack an in-depth understanding of the content and pedagogy (Ball, Lubienski, & Mewborn, 2001; Brown, Davis, & Kulum, 2011; Darling-Hammond & Sykes, 2003; Ojose, 2014; Shirvani, 2015). Therefore, the research represented by this paper aimed to examine the algebraic content and pedagogical knowledge of sixth grade mathematics teachers to identify their strengths and weaknesses in these areas through analysis of lesson plans, lesson notes, textbooks, and reference materials they used.

The purpose of this study was to examine algebraic content and pedagogical knowledge of sixth grade mathematics teachers through analysis of documents they used to determine their relative strengths and weaknesses to serve as the foundation for the development of teacher professional development curriculum. It is believed that the results of this study also could also contribute towards improving mathematics education in the country by informing teacher training institutions about the strengths and weaknesses with regard to teachers' knowledge of algebraic content and pedagogy, which could prove invaluable in designing their courses. Moreover, it is expected that the results of this study would contribute positively towards strengthening the national mathematics curriculum and the content development of prescribed textbooks and teacher resource books.

Theoretical Framework

The theoretical framework that guided this study is the domains of mathematical knowledge for teaching proposed by Ball, Thames, and Phelps (2008). The domains of mathematical knowledge for teaching was built on Shulman's theoretical framework connecting content knowledge to practice of teaching (Ball et al., 2008). According to Ball et al. (2008), in 1986 Lee Shulman and colleagues put forward a domain of teacher knowledge which linked content knowledge and teaching, and they called it pedagogical content knowledge. Since then, this domain has gained the popularity and Shulman's idea has been cited in more than 1,200 refereed journal articles in 125 different journals in "professions ranging from law to nursing to business, and regarding knowledge for teaching students preschool through doctoral studies" (Ball et al., 2008, p. 392).

Ball et al. (2008) proposed six domains, namely, common content knowledge; horizon content knowledge; specialized content knowledge; knowledge of content and students; knowledge of content and teaching; and knowledge of content and curriculum. These six domains come under two main categories – subject matter knowledge and pedagogical content knowledge.

Subject matter knowledge. Subject matter knowledge is the subject specific knowledge of content. This category is subdivided into common content knowledge, horizon content knowledge, and specialized content knowledge.

Common content knowledge. Common content knowledge is defined as "the mathematical knowledge and skill used in settings other than teaching" (Ball et al., 2008, p. 399). This includes knowing the materials they teach; being able to identify incorrect answers given by students; recognizing inaccurate definitions presented in textbooks; and being able to use terms and notations correctly. In short, common content knowledge is defined as "the mathematical knowledge known in common with others who know and use mathematics" (Ball et al., 2008, p. 403). Some research indicated that teachers' high school mathematics knowledge is positively correlated with students' learning gains (Shirvani, 2015).

Horizon content knowledge. Horizon content knowledge is defined as the awareness of how mathematical topics are related over the span of mathematics included in the curriculum (Ball et al., 2008, p. 403). For example, primary teachers may need to know how categorization of shapes and pattern generalizations can set the mathematical foundation for formal algebra, which is taught in sixth grade in the Maldives.

Specialized content knowledge. Specialized content knowledge is defined as "the mathematical knowledge and skill unique to teaching" (Ball et al., 2008, p. 400). An example of this could be the analysis of a student's incorrect answer to find out the nature of the error because, this would require dexterity in thinking about numbers, being aware of the patterns, and being able to critically examine the meaning in ways that are unique to teaching (Ball et al., 2008).

Pedagogical content knowledge. Ball et al. (2008) cited Shulman's definition of pedagogical content knowledge as follows:

> The most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations - in a word, the most useful ways of representing and formulating the subject that make it comprehensible to others. Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. (p. 391 – 392)

Magnusson, Krajcik, and Borko (as cited in Ball et al., 2008) defined pedagogical content knowledge as a teacher's understanding of how to help students understand specific subject matter. It includes knowledge of how particular subject matter topics, problems, and issues can be organized, represented and adapted to the diverse interests and abilities of learners, and then presented for instruction. The defining feature of pedagogical content knowledge is its conceptualization as the result of a *transformation* of knowledge from other domains (p. 394).

In short, pedagogical content knowledge is the unique knowledge required to teach subject-specific content. Ball et al. (2008) subdivided the pedagogical content knowledge into three domains. These domains are knowledge of content and students; knowledge of content and teaching; and knowledge of content and curriculum. Pedagogical knowledge of the teacher is important as teachers who have content knowledge but lack pedagogical knowledge not only find it difficult to explain mathematical concepts to the students in a way that they would understand but also are likely to make errors in classroom teaching (Ojose, 2014).

Knowledge of content and students. Knowledge of content and students is defined as the "knowledge that combines knowing about students and knowing about mathematics" (Ball et al., 2008, p. 401). This means teachers should be able to recognize how the students would think when presented with a certain problem and also teachers should be able to judge what the students would find confusing with the presented problem. In addition, teachers not only should be able to predict what kind of examples would make it easier for the students to grasp the concept but also what type of examples would maintain or build their interest in the lesson presented.

Knowledge of content and teaching. Ball et al. (2008) defined knowledge of content and teaching as "knowing about teaching and knowing about mathematics" (p. 401). In other words, teachers should be able to identify which instructional strategies would suit a certain lesson and also the sequence of the lesson, for instance, teachers being able to recognize which example would work best in the beginning or the introduction phase of the lesson.

Knowledge of content and curriculum. Ball et al. (2008) cited Shulman's definition of curricular knowledge instead of proposing a definition. Shulman defined curricula knowledge as

> [knowledge] represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances (as cited in Ball et al., 2008, p. 391).

Ball et al. (2008) developed the framework domains of mathematical knowledge for teaching after analyzing existing literature on knowledge base for teaching. This specifically focused on mathematics teaching and had identified the necessary aspects of mathematics knowledge for teaching.

The aim of this study was to examine algebraic content and pedagogical knowledge of sixth grade mathematics teachers through analysis of documents these teachers used, namely, lesson plans, lesson notes, textbooks, and reference materials.

Materials and Methods

Research Design

This study used a qualitative research design, specifically, document analysis method. Document analysis is a form of qualitative research in which documents are interpreted to give voice and meaning around a desired topic (Bowen, 2009). Bowen (2009) highlighted that document analysis is an important research tool. In document analysis it is better to gather as many documents as possible, however, it is also important to realize it is a quality not the quantity which matters (O'Leary, 2014). It is also vital to take into account the purpose of the document and target audience (Bowen, 2009). O'Leary's (2014) eight-step planning process was followed and the original documents were collected for the purpose of this study. In addition, producers of the documents were contacted to clarify doubts which arose at the time of document analysis to avoid personal bias and subjectivity.

The purpose of this study was to examine algebraic content and pedagogical knowledge of sixth grade mathematics teachers through analysis of documents these teachers used. According to Patton (2002), documents and written materials provide rich information and evidence in qualitative studies. Algebra lesson plans and lesson notes of the sixth grade mathematics teachers who consented to take part in this study were collected and analyzed to gain an in-depth understanding of the algebraic content and pedagogical knowledge of the sixth grade mathematics teachers. According to Chick, Pham, and Baker (2006), pedagogical knowledge of teachers could be assessed through teachers' lesson plans and lesson notes. The sixth grade algebra was divided into sixteen lessons and for each lesson it was expected that a separate lesson plan would prepared with lesson notes.

Setting and Sample

Participants of this study are in-service sixth grade mathematics teachers who were working in a similar environment with access to similar resources and trained from the same higher education institution in the Maldives. Teaching experience of the participants varied from 2-20 years. Sampling strategy used in this study was homogeneous, purposive sampling as this study aimed to algebraic content and pedagogical knowledge of sixth grade mathematics teachers through analysis of documents they used. In homogenous sampling the researcher purposefully selects individuals based on common characteristics (Creswell, 2012).

All the sixth grade mathematics teachers working in the selected schools were briefed about the research and only five teachers consented to take part in the study. Therefore, a total of 80 lesson plans and lesson notes were analyzed, in addition to the three prescribed textbooks which were *Mathematics in the National Curriculum Key Stage 2* (Naseer, Adam, & Shiunee, 2014) which gave detailed explanations to each of the strands in the curriculum with learning outcome indicators, *Exploring Mathematics 6 Teacher's Guide* (Adam & Shiunee, 2016a) which explained the pedagogy to be used in detail, and *Exploring Mathematics 6* (Adam & Shiunee, 2016b) which is the student book containing worked examples and explanations for students.

Data Collection and Analysis

Participants who consented to take part were requested to handover all the lesson plans, lesson notes, reference materials, and textbooks they used after each algebra lesson they taught. Data collection and data analysis was done concurrently. The doubts that arose while analyzing the documents were clarified by discussing with the respective teachers.

Content analysis was carried out to analyze the documents. According to Merriam (2009) all qualitative data analysis involves content analysis. Krippendorff (2003) stressed that content analysis is a scientific tool and defined content analysis as a "research technique for making replicable and valid inferences from texts (or other meaningful matter) to the contexts of their use" (p. 18). According to Bowen (2009), content analysis, then, is used as a "first-pass document review" (p. 32) that can provide the researcher a means of identifying meaningful and relevant passages.

The process of content analysis involved coding of raw data and categorizing them into themes according to characteristics of the content gathered. Bowen (2009) also notes thematic analysis, which can be considered a form of pattern recognition with the document's data. This analysis takes emerging themes and makes them into categories used for further analysis, making it a useful practice for grounded theory. It includes careful, focused reading and re-reading of data, as well as coding and category construction (Bowen, 2009). In this study, major themes were algebraic content knowledge and pedagogical knowledge and subthemes were the subdivisions in Ball and colleagues' (2008) model, namely, common content knowledge, horizon content knowledge, specialized content knowledge, knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum.

Results and Discussion

Documents and written materials such as lesson plans and notes can provide rich information and evidence in qualitative studies. Pedagogical knowledge of teachers could be observed through teachers' lesson plans and lesson notes (Chick, et al., 2006; Patton, 2002). Algebra lesson plans and lesson notes of the sixth grade mathematics teachers who consented to take part in this study were analyzed to gain an in-depth understanding of the algebraic content and pedagogical knowledge of the sixth grade mathematics teachers.

Lesson plans. Only one of the five teachers had a lesson plan. However, all the teachers had prepared some sort of lesson notes. Interestingly, it was the teacher who had two years teaching experience and that lesson plan was prepared when she taught mathematics for the first time which was two years ago. The lesson plan had three sections, namely, lesson starter, main activity, and plenary. Each of these sections had two parts, that is, teacher activity and student activity. The lesson plan was a one-page document and it lacked details.

Lesson starter. The teacher activity included greeting the students and introducing the lesson. Student activity was 'greet back' [teachers enter the class and say "good morning/ afternoon students" and the students are expected to say "good morning/ afternoon teacher"]. That was the only role students were expected to play in the "lesson starter" activity.

Main activity. The teacher activity was to explain examples on a certain page in the textbook. The student activity was to listen, clarify the doubts, doing the exercises on a given page, and give the students chance to present their answers. No other information was included.

Plenary. The teacher activity in this section read "summarize the lesson and check students' work" whereas student activity read "let them response to the teacher."

Four of the five teachers did not have a lesson plan. Some of them cited length of their teaching experience as the reason for not having a lesson plan while others said that they used the textbook so there was no need to have one.

Lesson notes. Analysis of lesson notes showed that they were just chunks taken from the prescribed textbooks indicating their over dependence on these prescribed textbooks. Lesson notes were mainly solutions to problems given in the student's book. Only the lesson plan and lesson note prepared to teach the first lesson included examples which were not in the prescribed textbooks. However, they followed the same pedagogy explained in the textbook (discussed below). For example, all the five teachers introduced algebra using what Tennant and Colloff (2014) referred to as 'fruit salad' approach to algebra. Two of the five teachers actually used fruits while one used fruits and vegetables, one used stationery (that is, books and pens), and the remaining teacher used front and the back of the teacher's hands to demonstrate like and unlike terms. This clearly indicated that the teachers lacked the common content knowledge as they were unaware that the "letters" or "variables" used in algebra represented numbers, not objects. Ball et al. (2008) placed common content knowledge as a subdivision of subject matter knowledge (content knowledge) and defined it as knowledge of mathematics that is common to everyone who knew and used mathematics.

Upon inquiry it became clear that those were "real-life" examples they used to introduce and relate the topic (algebra) to real-life. Participants believed that these real-life examples gave meaning to the topic and that helped to keep students' interest in the topic. To clarify, teachers were asked "why can't we write $3a+2b=5ab$" to which the teacher simply said that "you cannot write because they are unlike terms". In short, it became clear that teachers lacked pedagogical knowledge required to teach algebra. It is noteworthy that teachers' lack of algebraic content and pedagogical knowledge could lead students to develop misconceptions with regard to the concept of variables, making it difficult for students to understand problems involving algebraic expressions or equations (Ball et al., 2001; Koency & Swanson, 2000; Tajudin, 2014).

***Prescribed textbooks*.** Prescribed textbooks were specifically developed to cater the New National Key Stage 2 Mathematics Curriculum which was first implemented in 2016. This set of books contained three textbooks, namely, *Mathematics in the National Curriculum Key Stage 2* (Naseer, Adam, & Shiunee, 2014), *Exploring Mathematics 6 Teacher's Guide* (Adam & Shiunee, 2016a), and *Exploring Mathematics 6* (Adam & Shiunee, 2016b). Upon inquiry it became clear that it is mandatory that teachers used these prescribed textbooks and not any other materials to "ensure all the students across the country are exposed to the same learning experiences in terms of content and pedagogy" (S. Aishath, personal communication, May 29, 2018).

It is noteworthy that this set of books were developed specifically to cater the new national curriculum with the curriculum reform which was first implemented in 2015. In 2015 Foundation Stage (that is, pre-primary level) and Key Stage 1 (Grades 1, 2 and 3) was implemented, which was followed by Key Stage 2 (Grades 4, 5, and 6) in 2016. Key Stage 3 (which is Grades 7 and 8) was implemented in two phases, that is, phase 1 (Grade 7) was implemented in 2017 and phase 2 of Key Stage 3 (Grade 8) was implemented this January [January 2018] (S. Aishath, personal communication, May 29, 2018).

Mathematics in the National Curriculum Key Stage 2. All five teachers used Mathematics in the National Curriculum Key Stage 2 which gave detailed explanations to each of the strands in the curriculum with learning outcome indicators. Analysis of Mathematics in the National Curriculum Key Stage 2 revealed that this book was developed by a team of three curriculum developers from two institutions and eight mathematics teachers from eight different schools in the Maldives. This book included learning outcomes, indicators that showed the learning outcomes had been achieved in addition to teaching strategies. This is the prescribed textbook that was introduced with the New National Key Stage 2 Mathematics Curriculum which was implemented in 2016.

In-depth analysis of the topic Algebra showed that some of the explanations, suggested teaching strategies, and indicators were incorrect. This raised questions as to whether the team members were equipped with algebraic content and pedagogical knowledge required to develop such a book or teach algebra at sixth grade (or any grade for that matter).

Figure 1 and Figure 2 shows such examples taken from this book (Naseer, Adam, & Shiunee, 2014).

Topic: Algebraic Techniques	
Outcome PA2.1: *Model algebraic expressions and carryout addition and subtraction.*	**Indicators:** ***This is evident when the student:*** a. Models algebraic expressions using objects. Eg: %%%%= 4% !!!!!!!!= 8! b. Uses objects to add and subtract simple algebraic expressions. Eg: ff + fff = 2f + 3f = 5f

Figure 1. An incorrect indicator of learning (Naseer, Adam, & Shiunee, 2014, p. 78).

Topic: Algebraic Techniques	
Outcome PA2.1: *Model algebraic expressions and carryout operations. Write algebraic expressions and evaluate them by substitution. Simplify simple expressions. Solve simple linear equations.*	**Indicators:** ***This is evident when the student:*** a. Uses objects to add and subtract simple algebraic expressions. *(Eg: ff + fff = 2f + 3f = 5f; 11j – 7j = 4j; μμ+,,, = 2μ+3; È + ÈÈ – 7 = 3È – 7)* b. Adds and subtracts algebraic expressions involving 1-3 terms (exclude terms with exponents). *(Eg: w + 13 + w; 11s – 9t – 16s).*

Figure 2. An incorrect indicator of learning (Naseer, Adam, & Shiunee, 2014, p. 105).

The examples given in part (a) under indicators (both Figure 1 and Figure 2) are what Tennant and Colloff (2014) referred to as "fruit salad" algebra approach. This approach is one of the main factors contributing to the formation of misconceptions. Kuchemann (as cited in Welder, 2012) reported that only a few students between ages 11-13 were able to identify letters as a generalized number. The majority of the students identified variables as objects which is incorrect (Tennant & Colloff, 2014; Welder, 2012). For example, in part (a) it was also stated $!!!!!!!! = 8!$ which is incorrect as $8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40{,}320$ in mathematics. The example in the first part of (b) read $ff + fff = 2f + 3f = 5f$ which is mathematically incorrect as $ff = f^2$ and $fff = f^3$ in algebra (in the Maldives students will come across multiplication of like terms in grade seven). It is worth mentioning that when inquired how they explained multiplication of algebraic terms all five teachers said "multiply the numbers and write the letters next to the number" which is how it was explained in the lesson notes they used.

Unfortunately, these misconceptions go undetected at this level due the way mathematics curriculum is spread across the grades. It is also noteworthy that this book is used since 2016 with the implementation of new national curriculum, and this is a book written by a team of three curriculum developers and eight mathematics teachers. This not only raises serious questions regarding the state of the state of mathematics education in the country but also urgency in addressing the issue.

Exploring Mathematics 6 Teacher's Guide. All five teachers used Exploring Mathematics 6 Teacher's Guide, which explained the pedagogy to be used in detail for each of the topics and subtopics in the curriculum. Interestingly, the content of this book did not differ from what was explained in the Naseer, Adam, and Shiunee (2014). It was observed that greater emphasis was on procedural fluency than conceptual understating. Moreover, "fruit-salad" approach was used in explaining how teachers could relate the lesson to real-life. It became obvious from the lesson notes that teachers heavily relied on the prescribed textbooks in planning their lessons and making notes. Figure 3 shows an extract from this book which explained how the topic "algebra" can be introduced.

> **Pedagogy and Planning**
>
> The topic can be introduced in the class by showing some real life examples having pattern. For example, teacher can bring or flower to the class which has patterns. She can also show pictures of animals having pattern.

Figure 3. Suggested introduction to algebra (Adam & Shiunee, 2016a, p. 226)

From the analysis of documents, it became clear that teachers not only lacked algebraic content knowledge but also pedagogical knowledge. For example, all five teachers were unable to distinguish abbreviations from variables. They all used the "fruit-salad" approach to introduce algebra, indicating that all five teachers treated the variables as objects. It is important to highlight that teacher's lesson notes were a reflection of teacher's guide (Adam & Shiunee, 2016a). Teachers not being able to detect these incorrect explanations provide evidence that these teachers lack common content knowledge as well as knowledge of content and teaching (Ball et al., 2008). Ball et al. placed common content knowledge as a sub-category of subject matter knowledge (which in this study is algebraic content knowledge) whereas knowledge of content and teaching was placed under pedagogical knowledge. In short, this confirmed that teachers lack algebraic content and pedagogical knowledge required to teach at sixth grade.

Exploring Mathematics 6. All five teachers used Exploring Mathematics 6 which is the student book containing worked examples and explanations for students. Teacher's lesson notes made it clear that this is the main reference the teacher's used. Each of the students also had a copy of this book. This book offered little explanation of the topics and focused more on worked examples promoting procedural fluency over conceptual understanding. The explanation provided in the textbook did not include what the topic was about or why a certain approach was used. Instead, the book focused on explaining the steps that would guide students to obtain the correct answer to problems of similar kind. Moreover, the explanation offered with each step of the example showed how the steps were carried out instead of explaining or justifying the approach used in solving the problem. Furthermore, real-life applications of the topic were not included in the text. This could be the reason why teachers did not relate the topics to real-life or the misconception teachers had with regard to relating the topics to their real-life applications. For example, teachers thought using fruits or stationery to objectify variables was a real-life application of the topic. Figure 4 shows an extract from the book showing how the topic "algebraic techniques" is explained in the book and Figure 5 shows an example where a "skeleton" explanation is offered (Adam & Shiunee, 2016b).

Example 2: A fisherman has certain number of fishes in one box. He has 7 such boxes. He gave away three boxes of fish to his relatives. How many number of fish are left with him?

Remember that multiplication is repeated addition.

Answer:

Let the number of fish in one box be a.

Number of fish in 7 boxes $= 7a$

Number of fish he gave $= 3a$

Remaining fish $= 7a - 3a$

$= 4a$

Figure 4. Algebraic techniques explained (Adam & Shiunee, 2016b, p. 252).

Example 4: Simplify the following expressions.

a. $8(x-6)$ b. $-q(-2p-4r)$

Multiply each term inside the bracket with the variable or the number outside it.

Answer:

a. $8(x-6) = 8 \times x - 8 \times 6$

$= 8x - 48$

b. $-q(-2p-4r) = (-q) \times (-2p) + (-q) \times (-4r)$

$= 2qp + 4qr$

Remember, $(-) \times (-) = +$

Figure 5. "Skeleton" explanation (Adam & Shiunee, 2016b, p. 255).

It was also observed that the explanations provided for similar kind of questions were not consistent. It is not only important to maintain consistency in explanations to ensure students develop a firm understating of the procedures but also offer complete explanations and justify why a certain approach was used and why it works all the time. Incomplete or inconsistent explanations followed by dismissal of students concerns could lead to formation of misconceptions. As Naseer (2015) and Schnepper and McCoy (2013) stated, incomplete explanations and distorted definitions were the root cause of misconceptions. Figures 6 and 7 show extracts from the exercise section of the book where the variables were treated as objects, in other words, use of "fruit-salad" approach.

Mental Maths

1. Write the following as algebraic expressions.

   a. aaaa + nnnnnnn = ____________

   b. @@@@@@@@ − &&&&& = ____________

   c. EE + EEE − 8 = ____________

Figure 6. Use of "fruit-salad" approach in Mental Maths (Adam & Shiunee, 2016b, p. 253).

1. Write the algebraic expression for the following.

a. *********** = ____________

b. !!!!!! + ###### + :::::: = ____________

Figure 7. Use of "fruit-salad" approach in Exercise PA1.3 (Adam & Shiunee, 2016b, p. 257).

Analysis of exercise questions and examples given revealed that questions given in this book were very similar to the examples given in the book. Precisely, the difference between the examples and the exercise questions could be the use of a different a number or use of a different variable. For example, if how to expand $2(a+3)$ is explained, then in the corresponding exercise would include questions such as $4(a+3)$ and $2(w+3)$ for students to practice. The difference between the example $2(a+3)$ and the problem $4(a+3)$ is that the number 2 had been replaced with a 4 while, the difference between $2(a+3)$ and $2(w+3)$ is that the letter a had been replaced with the letter w. There were no questions that required students to put to use their critical thinking ability, analytical skills, or problem-solving strategies.

Analysis of lesson notes revealed that teachers followed the book in explaining the concepts to students. For example, teachers would state what the students were going to learn. After that the teacher would briefly explain the topic in two or three statements. This would be followed by various worked examples, indicating that they heavily depended on the textbooks. In summary, this textbook focused on explaining the procedures that would lead students to obtain the correct solutions to the problems given (which were of similar kind or pattern). The examples, explanations, and given questions did not promote conceptual understanding, critical thinking or even creativity. Lack of higher-order questions that required analytical skills and critical thinking made it difficult to identify whether students fully understood what was taught. Moreover, it hindered the opportunity to bring out misconceptions students might have.

Conclusion and Recommendation

Document analysis showed that the prescribed textbooks and other resource books prepared for the Maldivian schools had focused on computational procedures rather than developing conceptual understanding, problem-solving skills, or analytical skills. It also became clear that teachers were following the textbook and resource materials sent to them word for word. However, in order to promote knowledge growth and conceptual understanding it is important that explicit attention is given to differentiated instruction and open discussions rather than simple textbook exposure (Rittle-Johnson et al., 2011). Moreover, in-depth analysis of the textbooks and resource materials used by these teachers revealed that algebraic concepts were explained incorrectly in the prescribed textbooks, and resource materials. Teachers not being able to detect these incorrect explanations provide evidence that these teachers lack common content knowledge as well as knowledge of content and teaching (Ball et al., 2008). Ball et al. placed common content knowledge as a sub-category of subject matter knowledge (which in this study is algebraic content knowledge) whereas knowledge of content and teaching was placed under pedagogical knowledge. In short, this confirmed that teachers lack algebraic content and pedagogical knowledge required to teach at sixth grade.

An assumption of this study is that these teachers would have the same experience and exposure to mathematics content and pedagogy as they belong to similar kind of schools with similar resource allocations. A limitation of this study is that these findings cannot be generalized to the entire population. A delimitation of this study is that participants are trained from the same institution and belong to same nationality.

As teachers need a solid understanding of the concepts they teach so that they can teach for understanding by giving reasons for following certain procedure, it is recommended that mathematics teacher training programs emphasized the importance of teaching for understanding (Olanoff, Lo, & Tobias, 2014). It is also recommended that a study be carried out to assess the in-service teachers' content and pedagogical knowledge of algebra and based on the results a professional development program be developed to build on the strengths of the teachers and address the weaknesses of these teachers so that they are better equipped with the knowledge to teach mathematics for understanding. Last, but not least, it is recommended that the prescribed textbooks are revised urgently in the light of recent research and best and most effective ways of teaching algebra to ensure that a strong algebraic foundation is laid for the students which promotes not only procedural fluency but also conceptual understanding. Conceptual understanding has been associated with the way teachers introduced early algebraic concepts, which depends on the algebraic content and pedagogical knowledge of the teacher (Ball & McDiarmid, 1990; Chick et al., 2006; Tennant & Colloff, 2014).

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